Closing Thurs: 4.2
Closing Tues: 5.1/5.2 and 5.3

Today: Finish 4.2 and start 5.1/5.2

Entry Task: Your company makes two household cleaners: Miracle Bathtub Cleaner and Speedex Floor Cleaner. Your daily production of both cleaners combined is limited to 2,000 gallons. Your daily sales of Miracle Bathtub Cleaner never exceed 1,200 gallons, and your daily sales of Speedex Floor Cleaner never exceed 1,400 gallons.

CONSTRAINTS!

Finally, you make \$1.00 profit on each gallon of Miracle Bathtub Cleaner that you sell and \$2.00 on each gallon of Speedex Floor Cleaner that you sell. Determine the amount of each cleaner you should produce in order to maximize profit. ← OBJECTIVE!

x = GALLONS OF M.B.C.

y = GALLONS OF S.F.C

STEP 1: "…amount of each cleaner…"

x = "GALLONS OF M.B.C."

y = "GALLONS OF S.F.C.

	MBC	SFC
	x	y
PROFIT	\$1/gallon	\$2/gallon

STEP 2: Constraints and Objective?

$PROFIT = P(x,y) = 1 \cdot x + 2 \cdot y$

$P(x,y) = x + 2y$ ← OBJECTIVE

TOTAL GALLONS OF BOTH $= x + y \leq 2000.$

GALLONS OF M.B.C. $= x \leq 1200.$

GALLONS OF S.F.C. $= y \leq 1400.$

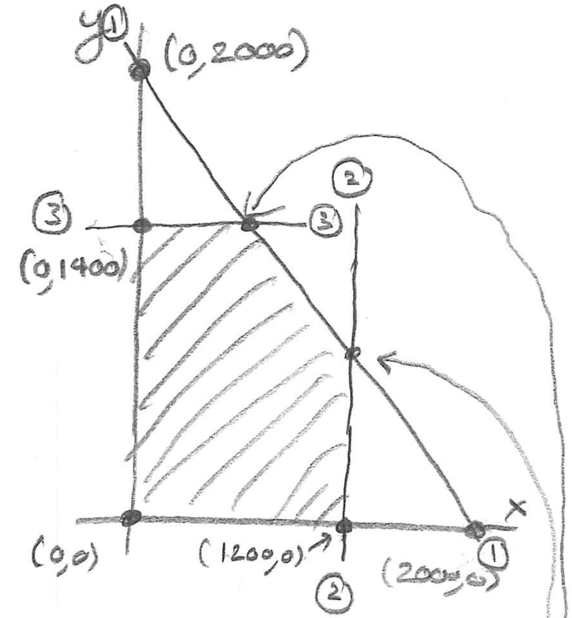
MAX PROFIT = \$3400
AND IT OCCURS
WHEN $x = 600$ GALLONS &
$y = 1400$ GALLONS

STEP 3: Graph

① $x + y = 2000$
(0, 2000) (2000, 0)

② $x = 1200$
(1200, 0) (1200, ANYTHING)

③ $y = 1400$
(0, 1400) (ANYTHING, 1400)



STEP 4: Corners?

5 CORNERS. (0, 0), (1200, 0), (0, 1400)

INTERSECTION OF ① $x + y = 2000$, ② $x = 1200$: $1200 + y = 2000$, $y = 800$

(1200, 800)

INTERSECTION OF ① $x + y = 2000$, ③ $y = 1400$: $x + 1400 = 2000$, $x = 600$

(600, 1400)

STEP 5: Evaluate objective

$(0,0) \to P(0,0) = (0) + 2(0) = \$0$

$(1200,0) \to P(1200,0) = (1200) + 2(0) = \$1200$

$(0,1400) \to P(0,1400) = (0) + 2(1400) = \$2800$

$(1200,800) \to P(1200,800) = (1200) + 2(800) = \$2800$

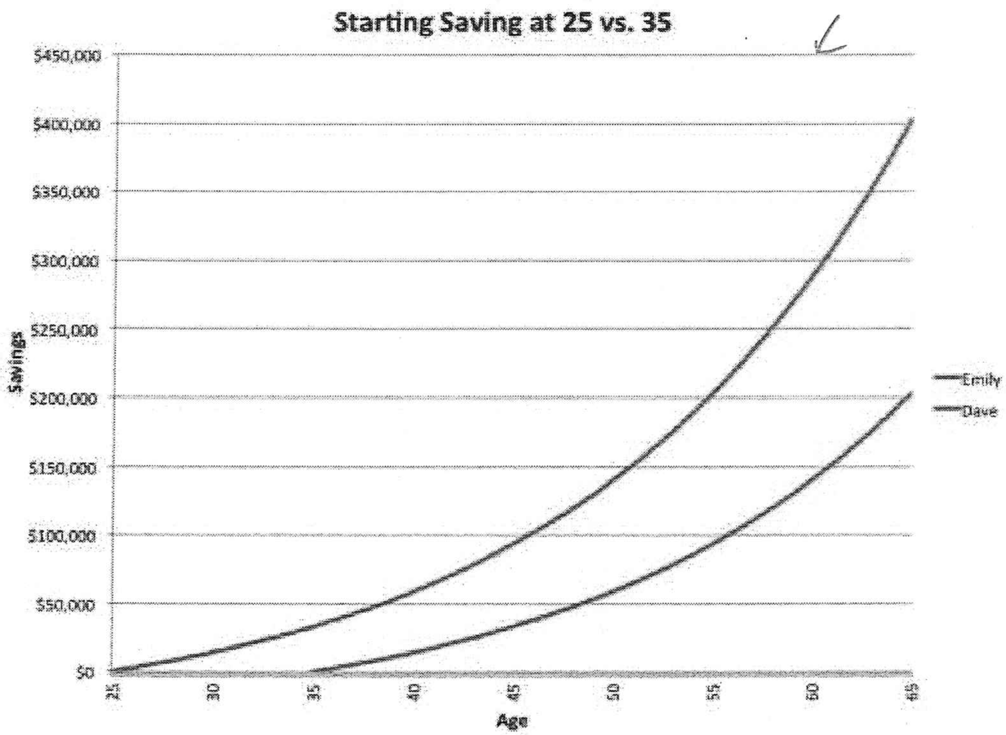
$(600,1400) \to P(600,1400) = (600) + 2(1400) = \boxed{\$3400}$

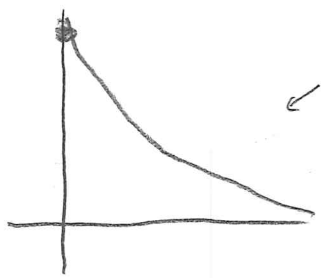
5.1/5.2 Exponentials & Logarithms

Exponential functions are everywhere!

Examples: Savings Accounts

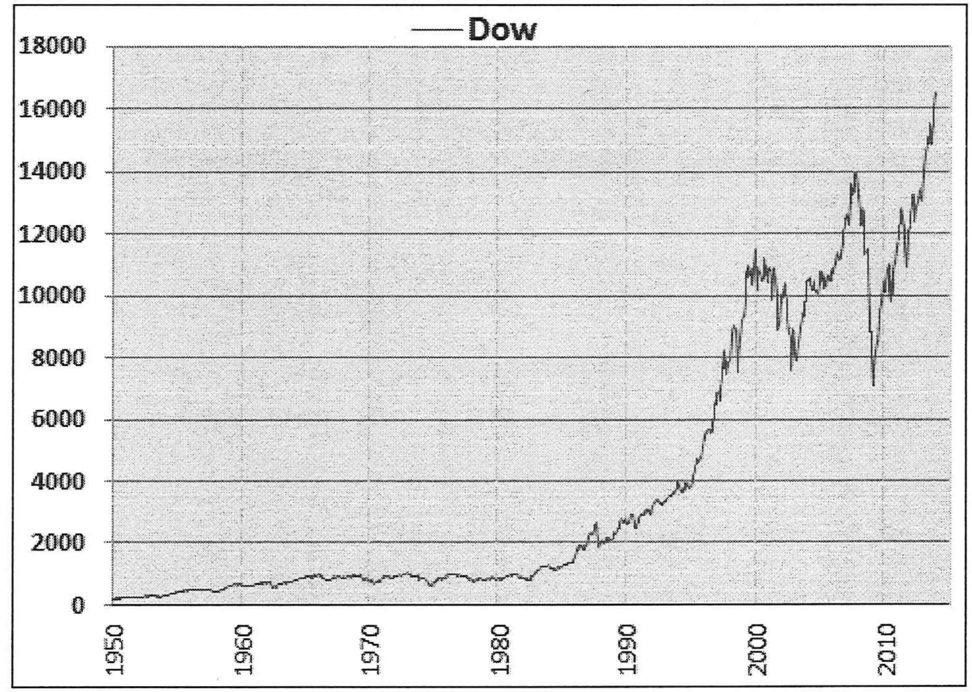
Exponential Growth





EXPONENTIAL DECAY

Economic Growth



Def'n: An exponential function can be written as

BASE

$$f(x) = A\, b^{x}$$

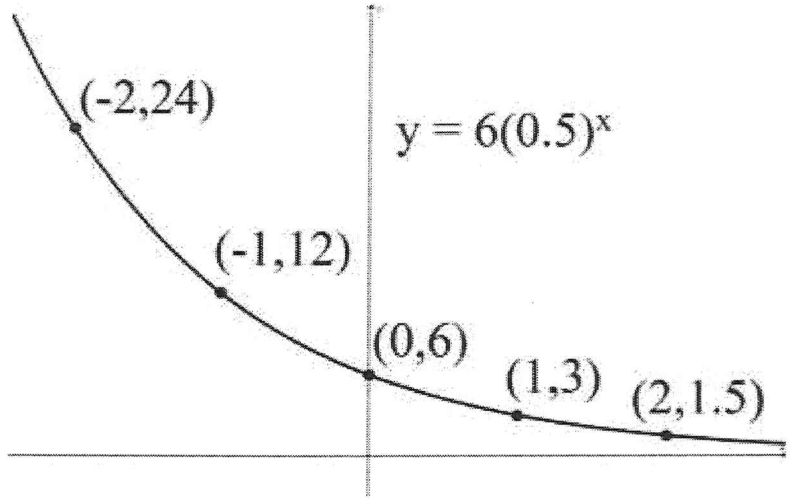
VARIABLE

A = *f(0)* = "the *y*-intercept"

b = "the base"

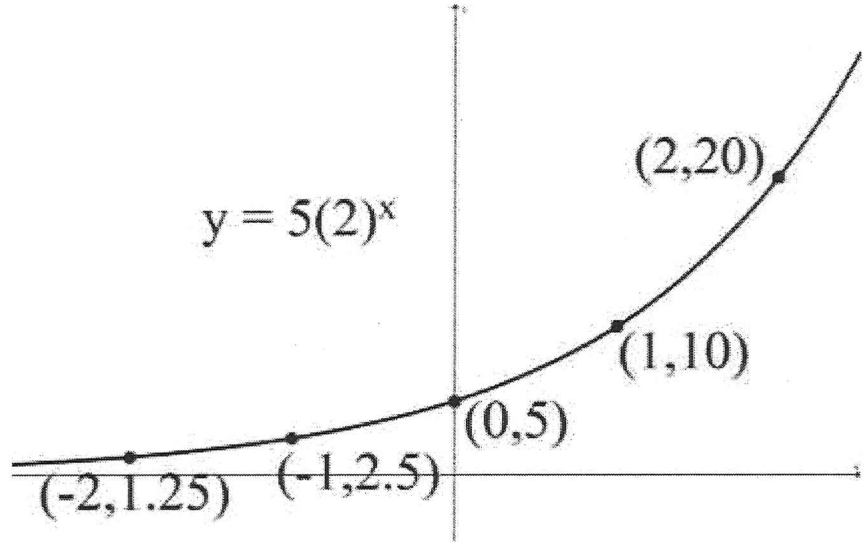
For $0 < b < 1$; exponential decay

Example: $f(x) = 6(0.5)^x$



If $b > 1$, exponential growth

Example: $f(x) = 5(2)^x$



Quick calculator practice:

- Plug $x = -2$ into $y = 6(0.5)^x$

$y = 6 * (0.5) \wedge (-2)$

$\boxed{y = 24}$

$y = 6\left(\frac{1}{2}\right)^x$

- Plug $x = 3$ into $y = 5(2)^x$

$y = 5 * \underbrace{(2) \wedge 3}_{8}$

$\boxed{y = 40}$

NOTE:

$5(2)^3 = 40$

IS NOT THE

SAME AS

$(5 \cdot 2)^3 = 1000$.

Euler's number

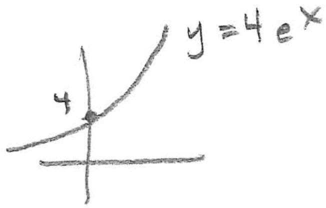
The number $e \approx 2.7182818245\ldots$ is called Euler's number and is the most often used exponential base in business, calculus and science (named after Leonard Euler a famous and prolific mathematician of the 1700's)

e^x is the same as

$(2.7182818245\ldots)^x$

WE WILL DISCUSS IN CH. 6 WHY WE USE THIS.

More calculator practice



1. Consider $y = 4\, e^x$
   - Plug x = 0 into $y = 4\, e^x$
   - Plug x = 2 into $y = 4\, e^x$
   - Exponential growth or decay?

$y = 4 \cdot e^0 = 4 \cdot 1 = 4$

$y = 4 \cdot e^2 \approx 29.5562244\ldots$

GETTING BIGGER, GROWTH!

$b = e \approx 2.71828182 > 1$

2. Consider $y = 4\, e^{-x}$
   - Plug x = 0 into $y = 4\, e^{-x}$
   - Plug x = 2 into $y = 4\, e^{-x}$
   - Exponential growth or decay?

$y = 4 \cdot e^{-0} = 4$

$y = 4 \cdot e^{-2} \approx 0.541341133\ldots$

GETTING SMALLER, DECAY!

$y = 4(e^{-1})^x \quad e^{-1} \approx 0.367879.. < 1$

Skills Review: *Power/Root/Exponent Facts*

Rule	Example	Example
$b^0 = 1$	$3^0 = 1$	$7e^0 = 7 \cdot 1 = 7$
$b^{\left(\frac{1}{n}\right)} = \sqrt[n]{b}$	$16^{1/2} = \sqrt{16} = 4$ $8^{1/3} = \sqrt[3]{8} = 2$ $9^{3/2} = \left(\sqrt{9}\right)^3 = 3^3 = 27$	$\sqrt[5]{e^x} = e^{x/5} = e^{\frac{1}{5}x}$
$b^{-x} = \dfrac{1}{b^x}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$	$\dfrac{1}{e^x} = e^{-x}$
$b^{x+y} = b^x b^y$	$x^2 x^3 = x \cdot x \cdot x \cdot x \cdot x = x^5$ $= x^{2+3}$	$e^x e^{3x} = e^{x+3x} = e^{4x}$
$\dfrac{b^x}{b^y} = b^{x-y}$	$\dfrac{x^8}{x^3} = \dfrac{x \cdot x \cdot x \cdot x \cdot x \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^5$ $= x^{8-3}$	$\dfrac{e^{5x}}{e^{2x}} = e^{(5x-2x)} = e^{3x}$
$(b^x)^y = b^{xy}$	$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x$ $= x^6 = x^{2 \cdot 3}$	$(e^3)^x = e^{3x}$

Solving with powers and roots

Powers/Roots:

$$y = x^n \leftrightarrow y^{(1/n)} = \sqrt[n]{y} = x$$

(Taking an even root? you need "±")

Get out your calculator

Solve

1. $x^2 = 7 \rightarrow x = \pm\sqrt{7}$
2. $\sqrt{y} = 3 \rightarrow y = 3^2 = 9$

3. $t^5 = 60 \rightarrow t = \sqrt[5]{60} = 60^{(1/5)} \approx 2.2679\ldots$
4. $\sqrt[5]{w} = 3 \rightarrow w = 3^5 = 243$

5. $\sqrt[3]{(2x-1)^5 - 5} = 3$

$(2x-1)^5 - 5 = 3^3 = 27$ ^3

$(2x-1)^5 = 27 + 5 = 32$ +5

$2x - 1 = (32)^{1/5} = 2$ ^(1/5)

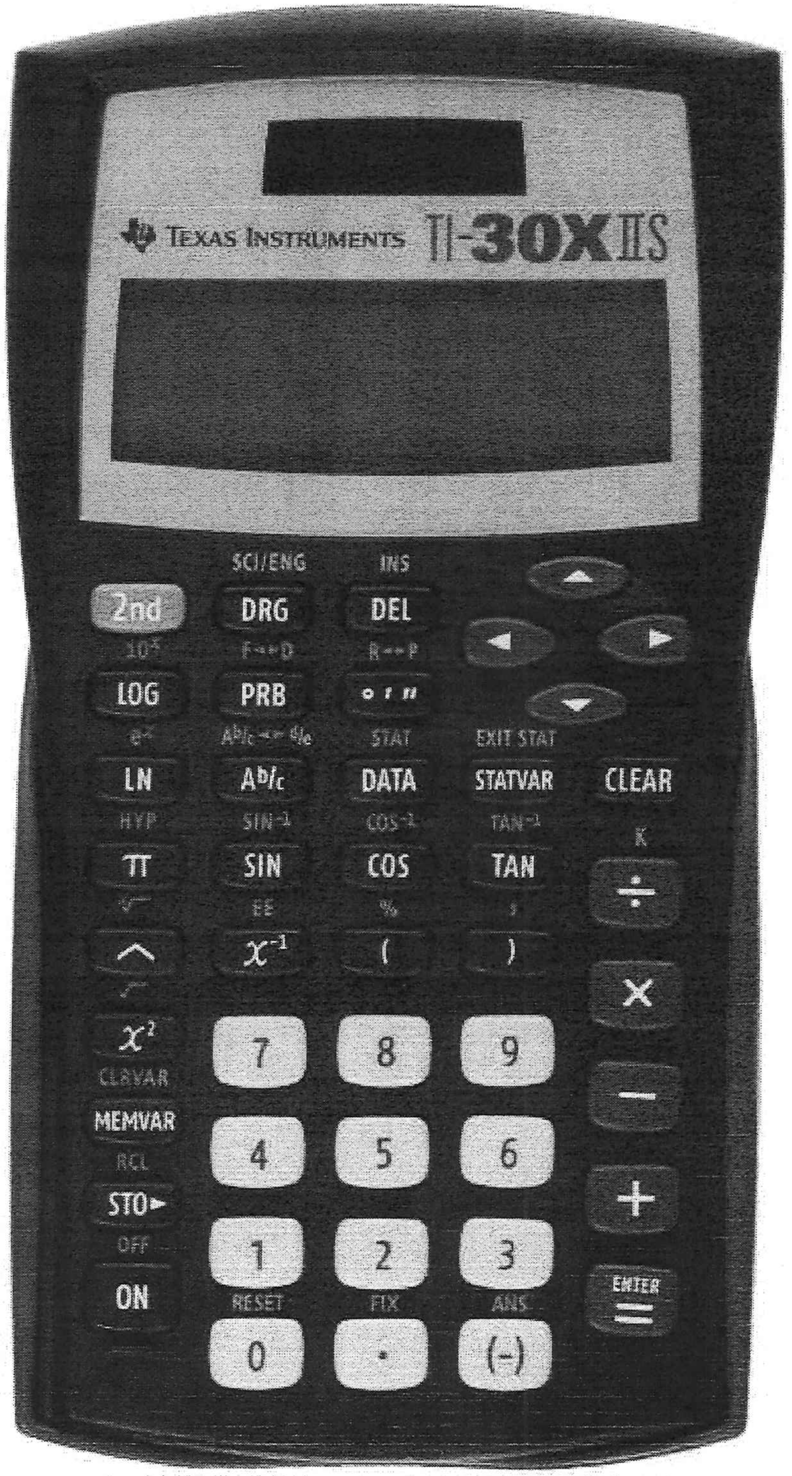
$2x = 2 + 1 = 3$ +1

÷2

$\boxed{x = \frac{3}{2} = 1.5}$

CHECK:

$$\sqrt[3]{\underbrace{\underbrace{(\underbrace{2\cdot(1.5) - 1}_{2})^5}_{32} - 5}_{27}} \stackrel{\checkmark}{=} 3$$



"$\sqrt{}$" → HIT $\boxed{2^{nd}}$, THEN $\boxed{x^2}$ ($\sqrt{}$)

"$\sqrt[5]{}$" → OPTION 1: TYPE $\boxed{5}$, THEN HIT $\boxed{2^{nd}}$

THEN HIT $\boxed{\wedge}$ ($\sqrt[x]{}$) ← $5\sqrt[x]{}$

→ OPTION 2: HIT $\boxed{\wedge}$, THEN $\boxed{(}$,

THEN TYPE "1/5", THEN $\boxed{)}$

LOOKS LIKE → ^(1/5)

"e^2" → HIT $\boxed{2^{nd}}$, THEN $\boxed{LN}$ (e^x), THEN TYPE 2.

LOOKS LIKE → e^(2)

"ln(14)" → HIT $\boxed{LN}$, THEN TYPE 14

LOOKS LIKE → ln(14).

Exponentials/Natural Logarithm

$$y = e^x \leftrightarrow \ln(y) = x$$
$$y = b^x \leftrightarrow \log_b(y) = x$$

Let's find some new buttons on your calculator (get your calculator out)

Example 1: Using your calculator:
STEP 1: Compute $e^2 \approx$ BLAH
STEP 2: Compute ln(BLAH) = ??

Example 2: Using your calculator:
STEP 1: Compute ln(3) = BLAH
STEP 2: Compute e^{BLAH} = ??

$\boxed{1}$ $e^2 \approx 7.389056099...$

$\ln(7.389056099) \approx 2$

$\boxed{2}$ $\ln(3) \approx 1.098612289...$

$e^{1.098612289} \approx 3$

Solve

1. $e^x = 7 \quad \leftrightarrow \quad x = \ln(7) \approx \boxed{1.9459101}$

2. $\ln(y) = 14 \leftrightarrow y = e^{14} \approx \boxed{1202604.284..}$

3. $e^{3x} = 4 \leftrightarrow 3x = \ln(4) \approx 1.386294361$

$x = \frac{\ln(4)}{3} = \frac{1.386294361}{3}$

$\boxed{x \approx 0.462098612}$

4. $2e^{10x} - 5 = 7$ $\quad +5$

$2e^{10x} = 12$ $\quad \div 2$

$e^{10x} = 6$ $\quad \ln(\ )$

$10x = \ln(6)$ $\quad \div 10$

$x = \frac{\ln(6)}{10}$

$x = \frac{\ln(6)}{10} \approx \frac{1.791759469..}{10}$

$\approx \boxed{0.1791759469}$

Review of your solving facts

Solve by using inverses in the correct order to get the variable by itself

Equation	Inverse
$x + 3 = 14$ $y - 5 = 22$	$x = 14 - 3 \quad = 11$ $y = 22 + 5 \quad = 27$
$3t = 16$ $\dfrac{m}{0.2} = 100$	$t = \frac{16}{3} \quad = 5.\overline{3}$ $m = 0.2 \cdot 100 \quad = 20$
$x^2 = 7$ $\sqrt{y} = 3$	$x = \pm\sqrt{7} \quad \approx \pm 2.64575...$ $y = 3^2 \quad = 9$
$t^5 = 20$ $\sqrt[5]{w} = 3$	$t = \sqrt[5]{20} \quad \approx 1.82056...$ $w = 3^5 \quad = 243$
$e^x = 10$ $\ln(y) = 3$	$x = \ln(10) \quad \approx 2.302585...$ $y = e^3 \quad \approx 20.0855369...$
$5^t = 60$	$t = \log_5(60)$

NOT IN CALCULATOR, WILL DISCUSS HOW TO COMPUTE NEXT TIME.

Logarithm Facts

Rule	Example
$1 = e^0$ and $\ln(1) = 0$ $e = e^1$ and $\ln(e^1) = 1$ $\ln(e^2) = 2$, $\ln(e^3) = 3$, and so on…	
$\ln(ab) = \ln(a) + \ln(b)$	$\ln(3) + \ln(5) =$ $\ln(3 \cdot 5) = \ln(15)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$	$\ln(20) - \ln(2) =$ $\ln\left(\frac{20}{2}\right) = \ln(10)$
$\ln(a^b) = b\ln(a)$	$\ln(2^x) =$ $x \cdot \ln(2)$ ← IMPORTANT!!!
$\ln(e^x) = x$ $e^{\ln(y)} = y$	$\ln(e^2) =$ 2 $e^{\ln(3)} =$ 3

Example:

Compute $\ln\left(\frac{e^3 e^4}{e^2}\right) = \ln(e^3 \cdot e^4) - \ln(e^2)$

$= \ln(e^3) + \ln(e^4) - \ln(e^2)$

$= 3 + 4 - 2 = \boxed{5}$